Math 3450 - Homework # 1 - Sets
Part A - Computations

Part 1 - Set builder notation

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1. Find all the elements from the set $\{n \in \mathbb{Z} \mid 1 \leq n^2 \leq 100\}$.
2. Let $S = \{1, 5, 7\}$ and $T = \{-1, 0, 10, 5\}$.
   Find all the elements in the set $X = \{a + b \mid a \in S, b \in T\}$.
3. Let $S = \{1, 5, 7\}$. Find all the elements in the set $Y = \{a^2 \mid a \in S\}$.
4. List all of the elements from $S = \{3k^2 + 1 \mid k \in \mathbb{Z} \text{ and } -1 \leq k < 4\}$
5. List 5 elements from the set $S = \{2x - 3y \mid x, y \in \mathbb{Z}\}$.
6. Use set-builder notation to write the set of all positive odd numbers.

Part 2 - Basic set operations

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7. Let $A = \{1, 5, -12, 100, 1/3, \pi\}$, $B = \{5, 1, -12, 18, -1/3\}$, $C = \{10, -1, 0\}$, $D = \{1, 2\}$, and $E = \{1, -1\}$. Calculate the following:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $A \cap C$
   (d) $A \cap \emptyset$
   (e) $B \cup \emptyset$
   (f) $D \times E$
   (g) $(D \cap A) \times (E \cup D)$
   (h) $C \times D$
   (i) $A - B$
   (j) $C - A$
   (k) $A - \emptyset$

(l) $A \cup B \cup C \cup D$

(m) $A \cap B \cap D$

(n) $A \cap B \cap C$

8. Let $A = \{1\}$. List the elements of the power set $\mathcal{P}(A)$.

9. Let $B = \{-1, 3\}$. List the elements of the power set $\mathcal{P}(B)$.

10. Let $C = \{2, 4, 6\}$. List the elements of the power set $\mathcal{P}(C)$.

Part 3 - Families of sets

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11. Let $A_n = \{x \in \mathbb{Z} \mid -n \leq x \leq n\}$.

    (a) List the elements in the sets A_1, A_2, A_3, and A_4.

    (b) Calculate $\bigcap_{n=2}^{\infty} A_n$ and $\bigcup_{n=5}^{\infty} A_n$.

12. Let $A_n = \{-2n, 0, 2n\}$.

    (a) List the elements in the sets A_1, A_2, A_3, and A_4.

    (b) Calculate $\displaystyle\bigcap_{n\in\mathbb{N}} A_n$ and $\displaystyle\bigcup_{n\in\mathbb{N}} A_n$.

13. In each of the following examples, the sets are intervals in the real line.

    (a) Let $A_n = (-n, n)$. Draw a picture of A_1, A_2, A_3, A_4.
    Then calculate $\displaystyle\bigcup_{n=1}^{\infty} A_n$ and $\displaystyle\bigcap_{n=1}^{\infty} A_n$.

    (b) Let $A_n = (1/n, 1)$. Draw a picture of A_2, A_3, A_4, A_5, A_6.
    Then calculate $\displaystyle\bigcup_{n=2}^{\infty} A_n$ and $\displaystyle\bigcap_{n=2}^{\infty} A_n$.

    (c) Let $A_n = (2 + 1/n, n)$. Draw a picture of A_3, A_4, A_5, A_6.
    Then calculate $\displaystyle\bigcup_{n=3}^{\infty} A_n$ and $\displaystyle\bigcap_{n=3}^{\infty} A_n$.

    (d) Calculate $\displaystyle\bigcup_{n\in\mathbb{Z}} (n, n+1)$ and $\displaystyle\bigcap_{n\in\mathbb{Z}} (n, n+1)$. Draw a picture.